16^{e} école d'été de didactique des mathématiques

Carcassonne, 21-28 août 2011

Thème 1

La profession d'enseignant de mathématiques, ses acteurs, ses problèmes…

et la recherche en didactique des mathématiques

Le calcul proportionnel et le symbole ∝

Enquête sur une œuvre mathématique méconnue

Séance de travaux dirigés conçue et animée par

Floriane Wozniak & Yves Chevallard

Mercredi 24 août 2011

➔ *On trouvera en italique, ci-après, des indications succinctes relatives aux consignes données aux participants quant aux tâches à accomplir et au rôle des documents présentés. Le texte qui suit n'est en aucune façon un compte rendu de séance.*

➔ *L'enjeu de l'activité est de faire rencontrer en situation la notion d'enquête.*

➔ *L'objectif du travail considéré est d'amorcer l'identification et l'étude d'une certaine* œuvre *O dont on ne connaît, au départ de l'enquête, que peu de chose.*

➔ *Au cours de la séance, les participants sont invités à accomplir deux tâches de types distincts : 1) rechercher (sur Internet) des documents proposant des « exposés » relatifs à l'*œuvre *O ; 2) examiner un petit corpus d'exposés issu d'une telle recherche et proposé par les animateurs de la séance.*

🡻

➔ *L'œuvre O a été mentionnée dans le cours donné par Y. Chevallard le mardi 23 août 2011. Le document ci-après permet de partir de cette évocation.*

Extrait du cours préparé par Yves Chevallard

« Une enquête cette fois plus incertaine – on recherche une œuvre dont la nature précise nous est inconnue et dont l'existence même est improbable – peut conduire à cette œuvre qu'on peut appeler le "calcul proportionnel", dont l'emblème pourrait être le symbole $\propto$ (que l'on trouve dans la police de caractères Symbol), et dont voici comment il permet d'alléger notablement et les calculs et la conduite des calculs. Tout d'abord on a $M \propto V = \frac{4\pi R^3}{3} \propto R^3$ et donc $v = \sqrt{2G\frac{M}{R}} \propto \sqrt{\frac{M}{R}} \propto \sqrt{\frac{R^3}{R}} = R$: la vitesse de libération est proportionnelle au rayon. Ensuite, on a $g = G\frac{M}{R^2} \propto \frac{R^3}{R^2} = R$: la pesanteur est elle-même proportionnelle au rayon. Bien entendu, un examen plus sévère de cette œuvre mathématique supposée serait alors de rigueur. En l'espèce, la séance 3 des travaux dirigés qui accompagnent ce cours sera consacrée à faire connaissance – jusqu'à un certain point − avec elle : connaissance praxéologique, bien sûr, mais aussi connaissance didactique, soit donc connaissance citoyenne, c'est-à-dire, ici, connaissance *professionnelle* – par et pour la profession. »

➔ *Dans une première partie de la séance, les participants s'efforcent, seuls ou à plusieurs, de rechercher sur Internet des éléments permettant l'identification et l'étude de l'œuvre O.*

➔ *Plus précisément, les questions auxquelles il convient de rechercher ainsi une réponse sont résumées ci-après.*

Enquête praxéologique & didactique

Qu'est-ce que c'est ? [Structure]
Comment est-on censé s'en servir ? [Fonctionnement]
Où cela vit-il, dans quelles institutions, chez quelles personnes ?
Quelles modes de diffusion ?
Y a-t-il des enseignements ?

🡻

➔ *La phase de recherche sur Internet commence alors ; chaque participant est invité à se donner les moyens de répondre, en fin de recherche, aux questions ci-après.*

Recherche sur Internet (ou ailleurs)

Quelles requêtes ? En quelles langues ?
Quels résultats ?

➔ *Un bilan est établi des difficultés rencontrées, des solutions trouvées, des résultats (conjecturaux) dégagés.*

➔ *On passe ensuite à la deuxième partie de la séance : l'examen et l'analyse d'un dossier constitué de documents qui disent quelque chose de l'œuvre O.*

➔

Un dossier

➔ Edward Batschelet, *Introduction to Mathematics for Life Sciences*, Springer-Verlag, Berlin, 1975, p. 71.

As an illustrative example we shall discuss the absorption of potassium (K) by leaf tissue of *Zea mays* (corn) as a function of time. We follow a report by Rains (1967). The independent variable is the time t, measured in hours. The dependent variable y is the amount of absorbed potassium, measured in µMoles per unit weight of leaf tissue (which is not specified here). The function $y = at$ fits the data very well for a domain $\{t \mid 0 \leqq t \leqq 4\}$. When the experiment is performed in darkness, the constant takes on the value $a = 1.8$ µMoles per unit weight per hour. If, however, the tissue is illuminated (by light intensity of roughly 2×10^4 lumen/m^2), then the constant turns out to be $a = 4.0$ µMoles per unit weight per hour. The constant a is called the *rate of absorption*. Hence the result of the experiment may be summarized as follows: The rate of absorption in the light is about twice the rate in the dark. The two straight lines are shown in Fig. 3.10.

Formula (3.5.1) means that y is proportional to x. For this relationship the symbol $\propto$ is sometimes used. Thus if the type of function is more important than the specific value of a, we write

$$y \propto x \tag{3.5.2}$$

and read "y is proportional to x"[4].

The constant proportion $y/x = a$ is called a *growth rate*, especially if the independent variable is time. Thus we speak of a rate of reaction, a rate of absorption, a rate of mutation, etc. In the Cartesian plane the constant a plays the role of a *slope*. If $a = 0$, the straight line coincides with the x axis. For $a > 0$ the straight line ascends from left to right and for $a < 0$ it descends. Some caution is required: A large value of a does not necessarily mean that the straight line is steep. All depends on the units we choose on the x and the y axes. The same linear relationship can be depicted in different ways. The two diagrams in Fig. 3.11 represent exactly the same functions as in Fig. 3.10, but in 3.11a the reader is under the impression that the two absorption rates are small, whereas Fig. 3.11b evokes the opposite impression. *The impression of steepness is highly subjective.* It is a well-known trick for advertisement or for political purposes to manipulate the steepness. A humorous as well as instructive example is presented in Huff (1954) and reproduced in Fig. 3.12.

In this connection we conclude that the *angle of inclination* of the straight line has no meaning in general. There are a few exceptions. In *calibration*, for instance, x means a variable measure read on the scale of an instrument, whereas y is the corresponding exact measure given

[4] The proportionality sign $\propto$ is very practical, but not widely used. Mathematicians apply it rarely. However, the symbol is quite popular in biophysics.

➔ « Proportionnalité », *Wikipédia*, 19 mai 2011.

http://fr.wikipedia.org/wiki/Proportionnalit%C3%A9

Proportionnalité

Ne doit pas être confondu avec proportionnelle.

On dit que deux mesures sont **proportionnelles** quand on peut passer de l'une à l'autre en multipliant ou en divisant par une même constante non nulle. Dans le cas où l'on multiplie, cette constante est appelée **coefficient de proportionnalité**.

Exemple : si, dans un magasin, le prix des pommes est de 2 euros le kg, il y a **proportionnalité** entre la somme S à payer et le poids P de pommes achetées, ce que l'on note parfois[1] :

$$S \propto P.$$

Le coefficient de proportionnalité est 2.

- Pour 1 kg, on doit payer 2 euros.
- Pour 3 kg, on doit payer 6 euros.
- Pour 1,5 kg, on doit payer 3 euros.

On remarque que le quotient des deux quantités est constant et est égal au coefficient de proportionnalité.

$$\frac{2}{1} = \frac{6}{3} = \frac{3}{1,5} = 2$$

Les Anciens comme Euclide auraient écrit que 2 est à 1 comme 6 est à 3 ou comme 3 est à 1,5.

➔ "Proportionality (mathematics)", *Wikipedia*, 20 juillet 2011.

http://en.wikipedia.org/wiki/Proportionality_%28mathematics%29

http://en.wikipedia.org/wiki/Proportionality_%28mathematics%29#Symbol

Symbol [edit]

The mathematical symbol '∝' is used to indicate that two values are proportional. For example, A ∝ B.

In Unicode this is symbol U+221D.

Direct proportionality [edit]

Given two variables *x* and *y*, *y is* ***(directly) proportional*** *to x* (*x and y* ***vary directly***, or *x and y are in* ***direct variation***) if there is a non-zero constant *k* such that

$$y = kx.$$

The relation is often denoted

$$y \propto x$$

and the constant ratio

$$k = y/x$$

is called the **proportionality constant** or **constant of proportionality**.

Examples [edit]

- If an object travels at a constant speed, then the distance traveled is proportional to the time spent traveling, with the speed being the constant of proportionality.
- The circumference of a circle is proportional to its diameter, with the constant of proportionality equal to π.
- On a map drawn to scale, the distance between any two points on the map is proportional to the distance between the two locations that the points represent, with the constant of proportionality being the scale of the map.
- The force acting on a certain object due to gravity is proportional to the object's mass; the constant of proportionality between the mass and the force is known as gravitational acceleration.

➔ Document Internet : http://aalem.free.fr/maths/mathematiques.pdf7

Signes et symboles mathématiques à employer dans les sciences physiques et dans la technique.

(extraits de la norme internationale iso 31-11 :1992)

Ce document regroupe des extraits choisis pour les élèves et les enseignants en CGPE de la norme internationale iso 31-11:1992 . Pour compléter cette norme, voici les symboles des sept unités de base :

nom	symbole
mètre	m
kilogramme	kg
seconde	s
ampère	A
kelvin	K
mole	mol
candela	cd

La valeur exacte de la vitesse de la lumière dans le vide est: $c = 2{,}997\ 924\ 58 \times 10^8\ \mathrm{m \cdot s^{-1}}$

Symboles divers

Symbole	Utilisation	Sens, énoncé	Remarques et exemples
$=$	$a = b$	a est égal à b	Le symbole $\equiv$ peut être utilisé pour souligner qu'une égalité est une identité.
$\neq$	$a \neq b$	a est différent de b	
$\stackrel{\text{def}}{=}$	$a \stackrel{\text{def}}{=} b$	a est égal par définition à b	$E_c \stackrel{\text{def}}{=} \frac{1}{2}mv^2$
$\triangleq$	$a \triangleq b$	a correspond à b	$1\ \mathrm{eV} \triangleq 11\ 604{,}5\ \mathrm{K}$
$\approx$	$a \approx b$	a est approximativement égal à b	Le symbole $\simeq$ est réservé pour « est asymptotiquement égal à »
$\propto$ ou $\sim$	$a \propto b$ ou $a \sim b$	a est proportionnel à b	
$<$	$a < b$	a est strictement inférieur à b	
$>$	$a > b$	a est strictement supérieur à b	
$\leqslant$	$a \leqslant b$	a est inférieur ou égal à b	

➔ Document Internet : http://whatis.techtarget.com/definition/0,,sid9_gci821576,00.html

proportionality

In mathematics, proportionality indicates that two quantities or variables are related in a linear manner. If one quantity doubles in size, so does the other; if one of the variables diminishes to 1/10 of its former value, so does the other.

The symbol for proportionality resembles a stretched-out, lowercase Greek letter alpha ($\propto$). When this symbol appears between two quantities or variables, it is read "is proportional to" or "varies in direct proportion with." Thus, the expression $x \propto y$ is read " x is proportional to y " or " x varies in direct proportion with y ." In this situation, as long as x and y do not attain values of zero, the quotient x / y is always equal to the same value k , which is called the proportionality constant.

➔ Document Internet :

http://uk.answers.yahoo.com/question/index?qid=20110115150459AAwXUxu

hbomb

Resolved Question Show me another »

What is the name of the proportionality symbol?

http://en.wikipedia.org/wiki/Proportiona...

where it says y is proportional to x, what is the symbol called?

7 months ago Report Abuse

Demiurge...

Best Answer - Chosen by Voters

It's called the proportional to symbol. There's no other name for it that I know of.

7 months ago Report Abuse

100% 1 Vote

Action Bar: Interesting! Email Comment (0) Save

This question about "What is the name of ... " was originally asked on Yahoo! Answers United States

Other Answers (1)

CogitoEr...

TOP CONTRIBUTOR

I call it Goldfish.

7 months ago Report Abuse

0% 0 Votes

➔ Document Internet :

http://rmarkmatthews.squarespace.com/storage/chemtutor/gas_law1.htm

The Ideal Gas Law

So what have we learned so far, gang? Well, we now have three spiffy little proportionalities:

$$V \propto 1/P$$

$$V \propto T$$

$$V \propto n$$

If we combine these into one equation, this gives us

$$V \propto (nT)/P$$

which we can rearrange into something that starts to look *vaguely* familiar.

$$PV \propto nT$$

Now remember how we set all those equations above to equalities by including a proportionality constant? Well, we do the same thing here.

$$PV = nCT$$

But you know, the letter C has been used to death in science, I mean you've got concentration, speed of light (which doesn't even start with c!), centi-. About we use a different letter for our constant. How about, oh, I don't know… the letter R.

$$PV = nRT$$

VIOLA!!!!!!

Here, it's useful to actually KNOW what our constant is (which is usually referred to as the molar gas constant), which has been calculated as 0.08206 L-atm/mol-K.

➔ *Correct use of "is proportional to" symbol (alpha)*

http://www.physicsforums.com/showthread.php?t=133911

• Checkfate [28 septembre 2006, 1 h 50]

Correct use of "is proportional to" symbol (alpha)

Hello, I am facing a problem that can be solved quite easily using the proportional symbol (I think), so I would like to try to use it! Only problem is.. I don't know exactly how to use it correctly...

The question is :**An astronaut weighs 882N on Earth, determing the weight of the astronaut on Planet X, which has a mass 95.3 times that of Earth and a radius 8.9 times that of Earth.**

So,

$$g = \frac{Gm}{r^2}$$

and thus

$$g\alpha \frac{m}{r^2}$$

So I wrote down

$$g\alpha \frac{m}{r^2}$$

$$g\alpha \frac{95.3}{79.21}$$

But of course this false... g is not proportional to 95.3/79.21.. lol. Can someone show me how to correctly show my work? Thanks. This would allow me to simply use this ratio to calculate his new weight.

• J77 [28 septembre 2006, 2 h 20]

Remember that G is the gravitational **constant**, ie. it always takes the value 6.67ishe-11

This constant turns the proportionality into an equality.

- Andrew Mason [28 septembre 2006, 2 h 56]

Andrew Mason

Posts: 4,978

Education: Industry Professional

Degree: B.A. (Math/Physics), LL.B. (law)

Recognitions:
Homework Helper
Science Advisor

Saying $g \propto m/r^2$ is equivalent to saying that $g = Gm/r^2$ where G is a constant (the proportionality constant) ie. g is a linear function of m and r^2. If you want to perform mathematical operations you have to use the equality sign and the constant.

$$g_1 = \frac{GM_1}{r_1^2}$$

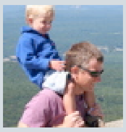
$$g_2 = \frac{GM_2}{r_2^2}$$

dividing, the constant falls out:

$$\frac{g_2}{g_1} = \frac{M_2}{M_1}\frac{r_1^2}{r_2^2}$$

- Chi Meson [28 septembre 2006, 4 h 34]

Chi Meson



Posts: 1,729

Degree: Bachelors in Physics, MFA in Writing, MAT in education

Recognitions:
Homework Helper

g is proportional to

$$\frac{m}{r^2}$$

but when cramming in the values you did, you get a comparison to Earth's "g." Multiply (95.3/79.21) by 9.81, and you get the "g" of the other planet.

• jpr0 [28 septembre 2006, 6 h 57]

If you want to use the proportionality sign, then say

$$g_e \propto \frac{M_e}{r_e^2} [/itex] and [tex] g_x \propto \frac{M_x}{r_x^2} [/itex] where [itex] g_{e/x} [/itex] refers to earth or planet x etc. Now you can say: [tex] \frac{g_x}{g_e} = \frac{M_x r_e^2}{r_x^2 M_e^2}$$

g_x)=g_e\frac{M_xr_e^2}{r_x^2M_e^2}

.

By the way, the "proprtional to" symbol isn't alpha. In tex it's "\propto"... heres the difference:

$$\alpha \ldots \propto$$

The first is alpha, the second is proptional to.

• Checkfate [28 septembre 2006, 12 h 24]

Thanks alot guys! :)

➔ *Combining Proportionality Statements*

http://www.physicsforums.com/showthread.php?t=510525

• prosteve037 [29 juin 2011, 2 h 41]

prosteve037

Posts: 27

Education: College Undergrad

Combining Proportionality Statements

If you have two statements, "a ∝ b" and "a ∝ c", you would get: "a = xb" and "a = yc" (where x and y are the constants of proportionality)

But what do you do so that it turns out to be "a = b × c"?

I've been searching for a DETAILED MATHEMATICAL explanation but have failed in finding one that responds thoroughly to my inquiry. I mean, I understand how it would result in "a = bc", but I'm having trouble understanding how to get there in concise mathematical steps and easy-to-understand mathematical reasoning.

• Unit [29 juin 2011, 13 h 58]

Unit

Posts: 164

Education: College Undergrad

Re: Combining Proportionality Statements

If anything, I would intuitively think that "a ∝ b" and "a ∝ c" imply "a ∝ bc", and thus a = kbc, where k is a constant of proportionality.

EDIT:
Actually, I'm wrong.

First, some definitions:
-to say of two variables *a* and *b* that "a ∝ b" means there exists a nonzero real constant *k* such that a = kb

"a ∝ b" and "a ∝ c" imply a = bk and a = jc for some nonzero k, j in R, respectively.
Then bc = (a/k)(a/j) = a^2/(kj) and so a^2 = (kj)bc, where (kj) is the product of two arbitrary constants in R and is thus itself an arbitrary constant in R. Therefore a^2 ∝ bc.

EDIT 2: And in fact, a = bc would imply that a^2 = (kj)bc = (kj)a, implying a = kj = constant. But a is a variable!

• mathman [29 juin 2011, 15 h]

mathman

Posts: 3,922

Recognitions:
Science Advisor

Re: Combining Proportionality Statements

This may be a quibble, but a=b x c means a ~ b where c is constant and a ~ c where b is constant.

- prosteve037 [30 juin 2011, 1 h 49]

prosteve037

Posts: 27

Education: College Undergrad

Re: Combining Proportionality Statements

Hmm. Is there no way to prove that "k = c" and "j = b" though?

- prosteve037 [10 juillet 2011, 19 h 03]

prosteve037

Posts: 27

Education: College Undergrad

Re: Combining Proportionality Statements

> Originally Posted by **mathman**
>
> *This may be a quibble, but a=b x c means a ~ b where c is constant and a ~ c where b is constant.*

I've been thinking about this and somehow it managed to clear things up a lot haha. Thank you.

But now there's something bothering me about this description; if you hold one variable constant, how can you say that that same variable is the constant of proportionality?

Looking at this now, this seems more of a general science question than it is a math question...

EDIT: What I mean is that given this description of what proportionality statements signify about dependent and independent/constant variables, do we just ASSIGN the constant variables to constants of proportionalities?

➔ Frank R. Giordano, William P. Fox, Maurice D. Weir, *A first course in mathematical modelling*, 2008.

http://books.google.com/books?id=XPxrItVsXpQC&pg=PP1&lpg=PP1&dq=A+first+course+in+mathematical+modelling+de+Frank+R.+Giordano,William+P.+Fox,Maurice+D.+Weir&source=bl&ots=4F4IiPLK-U&sig=xMNv4y7rE7Yf4slOURPGEydmZJA&hl=fr&ei=h2hNTr_3Csii8QP577HHBw&sa=X&oi=book_result&ct=result&resnum=3&ved=0CCsQ6AEwAjgK#v=onepage&q&f=false

2.2 Modeling Using Proportionality

We introduced the concept of proportionality in Chapter 1 to model change. Recall that

$$y \propto x \text{ if and only if } y = kx \text{ for some constant } k > 0 \tag{2.1}$$

Of course, if $y \propto x$, then $x \propto y$ because the constant k in Equation (2.1) is greater than zero and then $x = (\frac{1}{k})y$. The following are other examples of proportionality relationships:

$$y \propto x^2 \text{ if and only if } y = k_1 x^2 \text{ for } k_1 \text{ a constant} \tag{2.2}$$

$$y \propto \ln x \text{ if and only if } y = k_2 \ln x \text{ for } k_2 \text{ a constant} \tag{2.3}$$

$$y \propto e^x \text{ if and only if } y = k_3 e^x \text{ for } k_3 \text{ a constant} \tag{2.4}$$

In Equation (2.2), $y = kx^2$, $k > 0$, so we also have $x \propto y^{1/2}$ because $x = (\frac{1}{\sqrt{k}})y^{1/2}$. This leads us to consider how to link proportionalities together, a transitive rule for proportionality:

$$y \propto x \quad \text{and} \quad x \propto z, \text{ then} \quad y \propto z$$

Thus, any variables proportional to the same variables are proportional to one another.

Now let's explore a geometric interpretation of proportionality. In Equation (2.1), $y = kx$ yields $k = y/x$. Thus, k may be interpreted as the tangent of the angle θ depicted in Figure 2.8, and the relation $y \propto x$ defines a set of points along a line in the plane with angle of inclination θ.

Comparing the general form of a proportionality relationship $y = kx$ with the equation for a straight line $y = mx + b$, we can see that the graph of a proportionality relationship is a

➔ Document Internet : http://mathforum.org/library/drmath/view/62956.html

THE MATH FORUM **ASK DR. MATH**
QUESTIONS & ANSWERS FROM OUR ARCHIVES

Proportionality Symbol

```
Date: 05/20/2003 at 04:03:38
From: Lauren
Subject: Rates and variation symbol

Do you know the name of the little symbol used in the topic of rates
and variation that means "in proportion to"? It looks a bit like
alpha, only more (this sounds odd, but it's the best way to describe
it) "fish-like."
```

```
Date: 05/20/2003 at 12:06:00
From: Doctor Peterson
Subject: Re: Rates and variation symbol

Hi, Lauren.

Many symbols have no specific name, and are just referred to by their
usage; this can be called the proportionality symbol. In fact, the
'official' name of the symbol in Unicode (a set of international
characters for computers) is "proportional to":

   Symbol Characters and Glyphs - W3C Working Draft
   http://www.w3.org/Math/characters/html/symbol.html

Looking for a mention of an actual name, I looked up its history in
Jeff Miller's:

   Earliest Uses of Symbols of Relation
   http://jeff560.tripod.com/relation.html

It is just called "The symbol for variation (an eight lying on its
side with a piece removed)."

Eric Weisstein's World of Mathematics gives no name:

   Proportional
   http://mathworld.wolfram.com/Proportional.html

If you prefer to describe it by its appearance rather than strictly by
its usage, you might call it an "open alpha" or "loose alpha," rather
than "fishy alpha." People do often describe it (wrongly) as an alpha,
but I haven't seen these modifiers used anywhere.

If you have any further questions, feel free to write back.

- Doctor Peterson, The Math Forum
  http://mathforum.org/dr.math/
```

➔ Jeff Miller, *Earliest uses of symbols of relation*, 15 mai 2007
http://jeff560.tripod.com/relation.html

“The symbol for variation (an eight lying on its side with a piece removed) was introduced in 1768 by W. Emerson in *Doctrine of Fluxions* (3d ed., London) (Cajori vol. 1, page 297).”

➔ Florian Cajori, *A history of mathematical notations*, vol. 1, 1928, p. 297
http://ia700506.us.archive.org/9/items/historyofmathema031756mbp/historyofmathema031756mbp.pdf

A special symbol for variation sometimes encountered in English and American texts is $\propto$, introduced by Emerson.[4] “To the Common Algebraic Characters already receiv'd I add this $\propto$, which signifies a general Proportion; thus, $A \propto \frac{BC}{D}$, signifies that A is in a constant ratio to $\frac{BC}{D}$.” The sign was adopted by Chrystal,[5] Castle,[6] and others.

[4] W. Emerson, *Doctrine of Fluxions* (3d ed.; London, 1768), p. 4.
[5] G. Chrystal, *Algebra*, Part I, p. 275.
[6] Frank Castle, *Practical Mathematics for Beginners* (London, 1905), p. 317.

➔ William Emerson, *The Doctrine of fluxions*, 2e éd., 1757, p. 4
http://books.google.com/books?id=xg8OAAAAQAAJ&printsec=frontcover&hl=fr&source=gbs_ge_summary_r&cad=0#v=onepage&q&f=false

4. To the common Algebraic Characters already receiv'd I add this $\propto$, which ſignifies a general Proportion; thus, $A \propto \frac{BC}{D}$, ſignifies that A is in a conſtant Ratio to $\frac{BC}{D}$; that is (if a, b, c, d be other Values of theſe Quantities) $A : \frac{BC}{D} :: a : \frac{bc}{d}$; and thus every general Proportion is to be underſtood.

➔ George Chrystal, *Algebra. An elementary text-book*, vol. 1, 1866, pp. 273-279
http://djm.cc/library/Algebra_Elementary_Text-Book_Part_I_Chrystal_edited.pdf

When y depends on x in the manner just explained it is said *to vary directly as* x, or, more shortly, *to vary as* x.

A better * phrase, which is also in use, is "*y is proportional to x.*"

This particular connection between y and x is sometimes expressed by writing

$$y \propto x.$$

§ 19.] In place of x, we might write in equation (2) x^2, $1/x$, $1/x^2$, $x+b$, and so on ; we should then have

$$y = ax^2 \quad (\alpha),$$
$$y = a/x \quad (\beta),$$
$$y = a/x^2 \quad (\gamma),$$
$$y = a(x+b) \quad (\delta).$$

* The use of the word "Variation" in the present connection is unfortunate, because the qualifying particle "as" is all that indicates that we are here concerned not with variation in general, as explained in § 17, but merely with the simplest of all the possible kinds of it. There is a tendency in uneducated minds to suppose that this simplest of all kinds of functionality is the only one ; and this tendency is encouraged by the retention of the above piece of antiquated nomenclature.

À suivre…

➔ Frank Castle, *Practical mathematics for beginners*, 1905, p. 317
http://ia600402.us.archive.org/27/items/practicalmathema00castrich/practicalmathema00castrich.pdf

6. In any class of turbine if P is power of the waterfall and H the height of the fall, and n the rate of revolution, then it is known that for any particular class of turbines of all sizes

$$n \propto H^{1 \cdot 25} P^{-0 \cdot 5}.$$

In the list of a particular maker I take a turbine at random for a fall of 6 feet, 100 horse-power, 50 revolutions per minute. By means of this I find I can calculate n for all the other turbines of the list. Find n for a fall of 20 feet and 75 horse-power.

➔ *Nouvelle encyclopédie autodidactique Quillet. L'enseignement moderne et pratique*, publiée en collaboration par un comité d'universitaires, 1958, tome 1, pp. 216-217.

202. GRANDEURS PROPORTIONNELLES À PLUSIEURS AUTRES

Il peut arriver qu'une grandeur A dépend proportionnellement de plusieurs autres.

Ainsi le poids d'un cylindre métallique varie avec la densité du métal, le rayon de sa base, la longueur de son axe ; son volume varie avec son rayon et la longueur de l'axe.

1° Une grandeur A est directement proportionnelle à plusieurs autres B, C, D, E si ces dernières grandeurs sauf une E, restant fixes, les valeurs de A et de E varient dans le même rapport.

2° Une grandeur A est directement proportionnelle à la grandeur B et inversement proportionnelle à la grandeur C lorsque :

a) C étant constante, les valeurs de A et B sont directement proportionnelles ;

b) B étant constante, les valeurs de A et C sont inversement proportionnelles.

203. Soient : V le volume d'un cylindre de hauteur *h* et de rayon *r*.

V_1 le volume d'un autre cylindre de hauteur *h* et de rayon *r'*.

V' le volume d'un troisième cylindre de hauteur *h'* et de rayon *r'*.

La longueur de l'axe ne variant pas, les volumes sont directement proportionnels aux carrés des rayons :

$$\frac{V}{V_1} = \frac{r^2}{r'^2}$$

Les rayons ne variant pas, les volumes sont directement proportionnels aux longueurs des axes :

$$\frac{V_1}{V'} = \frac{h}{h'}$$

Multiplions membre à membre ces deux égalités :

$$\frac{V}{V_1} \times \frac{V_1}{V'} = \frac{r^2}{r'^2} \times \frac{h}{h'}$$

ou :

$$\frac{V}{V'} = \frac{r^2}{r'^2} \times \frac{h}{h'} = \frac{r^2 \times h}{r'^2 \times h'} ;$$

d'où le théorème :

Théorème. – Si une grandeur A est directement proportionnelle à plusieurs autres, les valeurs de A sont directement proportionnelles aux produits des valeurs correspondantes des autres grandeurs.

➔

➔ *On a listé ci-après quelques URL de documents qu'il serait possible d'examiner mais qui ne le seront pas, en principe, dans le cadre même de la séance.*

Un corpus complémentaire (dans un ordre quelconque)

1. http://fr.wikiversity.org/wiki/Apprendre_%C3%A0_lire_les_expressions_math%C3%A9matiques/Symboles_de_types_lettres

2. http://www.mathmotivation.com/lectures/Variation.pdf

3. http://forums.futura-sciences.com/astronomie-astrophysique/105641-equations-de-friedmann-lemaitre-aux-graphes-maple-attention-maths-inside.html

4. http://mathias.bavay.free.fr/these/html/node312.html

5. http://physique.unice.fr/optique/coh/cspat/node9.html

6. http://www.ericweisstein.com/research/thesis/node22.html

7. http://ebookbrowse.com/intro-p1-lp104-pdf-d49724662

➔

➔ *La séance s'achève sur un petit travail de synthèse qui devrait permettre de nourrir un compte rendu de ce début d'enquête.*

Rapport d'enquête