GÉRARD SENSEVY, MARIA-LUISA SCHUBAUER-LEONI, ALAIN MERCIER, FLORENCE LIGOZAT and GÉRARD PERROT

AN ATTEMPT TO MODEL THE TEACHER'S ACTION IN THE MATHEMATICS CLASS

ABSTRACT. This paper outlines some theoretical categories (i.e. the *meso-, topo-, chrono-geneses*, the "milieu", the didactical contract and the learning games), providing a *model* to study mathematics teacher's action. In order to show what this model brings to the didactical analysis, we present the action of two teachers, on the same content, and we attempt a *threefold description*, covering different scales of analyses of the teaching processes. To amplify the phenomena that are to be observed, we suggested the teachers include the "Race to 20" situation in their teaching. We expect that implementing an unusual teaching device should lead the teachers to take decisions and explain them more easily than in everyday lessons.

KEY WORDS: chronogenesis, didactical contract, mathematic situation, mesogenesis, milieu, teacher's action; race to 20, threefold descriptive model, topogenesis

1. Introduction

We study mathematics teachers' actions within a model that attempts to connect and enhance several theoretical frameworks, borrowed mainly from TDS, the theory of didactic situations, and from ATD, the anthropological theory of didactics.

Our approach is resolutely nonprescriptive; it consists in describing the interaction of a teacher and his students in order to improve our understanding while respecting the complexity of the teaching process. In Section 2 we provide a brief description of the context of the research and we define the categories we find necessary to model the teacher's action. Section 3 is devoted to a description of the empirical set-up of the model. The conclusion gives possible ways of continuing this research and points out the new implications of this type of work for teacher training.

2. The framework of the research

2.1. *Present work*

The general aim is to describe and understand the teacher's action in the mathematics class from a didactical point of view. We consider that the

Educational Studies in Mathematics (2005) 59: 153–181
DOI: 10.1007/s10649-005-5887-1 © Springer 2005

pieces of knowledge at stake affect the teacher–students interactions, in such a way that the teaching and learning processes cannot be studied separately from the mathematical subject. Therefore, we assume that *looking at the interaction patterns through classroom observations* may enable us to see what is, in fact, taught and especially *how it is taught*. More precisely, we are looking at *teaching techniques* that could be specific of the teacher's action in the mathematics class. These techniques, that may be classified into categories, mainly born from TDS and ATD (see Section 2.2), are to be the core of our teacher's work model. However, we will not neglect the fact that the teacher's acts may also be related to more general groundings in education and learning theories. This aspect will lead us to *take into account the teacher's comments* about his own action during the class obtained in interviews (see Sections 2.1 and 2.3). Beyond the spotting of the techniques, the links between the different categories of action should be showed in order to reveal how our model is being elaborated.

In this paper, we shall consider the teacher's action while he[1] has to carry out a "Race to 20" lesson, suggested by the research team, adapted from Brousseau's situation (Brousseau, 1997, pp. 3–17). This situation is used as a paradigm for studying the didactical action based on the classroom interactions, and as a means to improve our theoretical framework. Indeed, the structure of the "Race to 20" situation, should enable a wide range of actions to occur—both on the teacher's side and the students' side. We assume that most of these actions are induced by the mathematical features of this situation.

2.1.1. *What is the "Race to 20" situation?*

The situation is based upon a game which opposes two players. The first player says a natural number X_1 that is less than 3 (1, for example). The second player says a natural number Y_1 obtained by adding 1 or 2 to X_1 (for example, he says 3, a number obtained by adding 2 to 1). The first player then says a natural number X_2, obtained by adding 1 or 2 to Y_1 (for example, he adds 1 and says 4), etc. The player who is the first to say 20 is the winner.

There are numbers that it is sufficient to say in order to win: 2, 5, 8, 11, 14, 17, 20. However most students do not spontaneously understand this. They quite quickly figure out that 17 is a winning number[2] but they need to play many rounds of the game to find out that so is 14. Therefore, the teacher's intervention is necessary to enable the students to discover the role of these numbers in the game. Brousseau (1997) shows how didactic engineering about the "Race to 20" led to the discovery of the didactic conditions, in which students come up with theorems such as "17 wins, so the 'Race to 20' equals to the Race to 17," then, "14 wins, ... " and so on, followed by the notion of "winning strategy" and that of "game equivalence." "Euclidean division" is a general model of this type of game, i.e. reaching a number $A (A = 20$ in the "standard" race to 20) in an integer number Q of steps of length $B (B = 3$ in the "standard" race to 20), giving the following equation $A = B * Q + R$, where R is the remainder in division of A by Q, and relatively to the game, the number to start with in order to win.[3] According to Brousseau (1997, p. 3) the "Race to 20" is a situation "to revisit division (in circumstances in which the 'meaning' of the operation did not conform to the one learned earlier)." In practice, however, this approach is rarely observed.[4]

In fact, Brousseau's situation has another aim: "foster the discovery and demonstration, by the children, of a sequence of theorems" (ibid., p. 4). For that purpose, the situation is divided in three phases in Brousseau's engineering: a phase of action (playing one-against-one); a phase of formulation (playing team-against-team), in which "the teacher nominates one child as the team representative for each round, naming her at random (ibid., p. 4)"; a phase of validation (the game of discovery): the students "have to put forward propositions and to prove to an opponent that they are either true or false" (ibid., p. 4).

It is the design and study of the Race to 20 situation, that led Brousseau to the general concept of didactic situations and their classification into situations of "action", "formulation" and "validation". This is a strong reason to choose this didactic setup as a paradigm for the studying of teacher's work. Indeed, this situation can be regarded as a very appropriate one to understand the teacher's action (or lack of action) through the different dialectics of action, formulation, validation (Brousseau, 1997, p. 9–11) that it requires.

2.1.2. *The research organization and discussion*

The research involved two teachers (T1 and T2) of grade V classes in an elementary school.

We first introduced the teachers to the "Race to 20" situation, presenting the main mathematical aspects of the game in a 2 hours training. Brousseau's complete text on this subject was handed out at the end of the training session, but the didactic engineering itself was not a training topic during this session. Using this engineering in the teaching process was not compulsory, nor was the reading of the text. It was given as an opportunity that the teachers could take into account, or not. As we will see below, they used this possibility in different ways.

- In the second phase, we asked each teacher to teach one or more lessons on the situation "Race to 20." The teachers were free to plan the lessons as they wished. Both decided to devote two lessons to the situation. We conducted interviews with the teachers before and after the lessons.
- The third phase consisted in the teachers' self-analyzing their first lesson, based on a video recording of the class.

All the lessons were audio- and videotaped and verbal interactions were transcribed from an audio source mainly but including details from the pictures, like words on the blackboard, when needed.

In this paper, our analysis focuses mainly on the first lesson, in order to concentrate our descriptions on the fundamental techniques the teachers used. In this research, we tried to obtain a synthesis of studies of "ordinary classes in everyday conditions" and those based on teaching experiments based on "didactical engineering." This synthesis of clinical and experimental approaches refers to the *ad hoc* theoretical and empirical approaches described in Leutenegger (1999) and Schubauer-Leoni and Leutenegger (2002).

In our research, the experimental perspective is involved in proposing a situation (here, "Race to 20") to teachers who had never tried it before. First of all, because Brousseau's situation assumes a large spectrum of teaching actions concerning the action, formulation and validation phases management, we expect the situation to reveal many of the fundamental teaching techniques at work in the didactic process. Furthermore, we think the novelty of this situation for the teachers should instigate or concentrate in a short time some usual techniques that normally occur in every didactic process.

The clinical dimension relies, first of all, upon the relative freedom the teachers had been given in organizing the lessons based on the "Race to 20" situation. Indeed, they could use their "ordinary" teaching techniques, to some extent. In this case, the clinical study of didactic systems is meant to follow the dynamics of the usual didactic processes at work during the lesson. Moreover, beside the meanings inferred from the observation of the classes, we also took into account those emerging from the teacher's self-analyses, while watching the video recordings of their lessons.

In this paper, we do not take into account all aspects of our research: our specific aim is only to demonstrate how some theoretical categories can be used in the description of teaching processes.

2.2. *The theoretical framework*

To analyze the teacher's action, we need to describe this action by using categories specific to didactic interaction.

2.2.1. *The didactic relationship*

The didactic relationship is a ternary relation between the teacher, the students, and the pieces of knowledge at stake. We assume that the teacher's work consists in initiating, establishing, and monitoring this relationship. Using the concept of didactic relationship is a way to emphasize the communicative nature of teaching techniques, and to focus on the fact that the core of the relationship between the teacher and the students is their sharing of this piece of knowledge. We consider that the didactic relationship is fundamentally threefold: understanding between the teacher and his students thus implies not only analyzing their respective positions but, especially, taking into account the knowledge that will be the focus of the lesson.

2.2.2. *The adidactic situation*

An adidactic situation is a learning environment designed by the teacher. We need two criteria to design and to understand such a situation.

First, the student should not be aware of the teacher's intentions about the knowledge underlying the situation.

Second, the student is engaged in a game, "this game being such that a given piece of knowledge will appear as *the* means of producing winning strategies" (Brousseau, 1997, p. 7). "The set of constraints and resources available in this game (situation) allows and directs students' adidactic action. This set is named *milieu*" (Brousseau, 1997, p. 248).

In our research, the milieu is made of the rules of the "Race to 20", which is an adidactic situation. The students' interactions with the milieu are supposed to be sufficiently "significant and adequate" to enable them, case by case, to gain knowledge, to formulate strategies of action or validate their understandings (second criterion). When students use the feedback coming from these milieus, their activity is not influenced by the necessity to satisfy the assumed expectations of the teacher (first criterion). One must note that the milieu, as a "set of constraints and resources" includes material objects (e.g. the writings on the board or the students' notebooks) as well as symbolic objects (e.g. the rules of the game, and also the successive "theorems" produced by the students).

2.2.3. *The didactic contract*

The teacher has to monitor the students' activity and the associated learning, by handling the evolution of the situations and of their milieus. By doing so, he defines the didactic contract that governs the didactic relationship and defines the conditions of its existence.

Depending on the point of view one adopts, the didactic contract appears to the observer as a set of reciprocal expectations between the teacher and the students. As Brousseau (1997, p. 54–58, 225) says: "[the] (specific) habits of the teacher are expected by the students and the behaviour of the student is expected by the teacher; this is the didactic contract." These expectations can be viewed as a set of largely implicit rules, of usual ways of acting (with regard to the subject being studied) that the teacher and the students find suitable in the context of the didactic relationship. Some of these habits are perennial (Mercier, 1988), and we consider these to be the basis of the didactic relationship. Others are specific to the concept being taught, and therefore depend on the evolution of the milieu during the lesson.

2.2.4. *Specific and generic techniques*

To understand the teacher's action, we have to describe the techniques that he produces. Some of these techniques are specific to each piece of knowledge. In the "Race to 20", for example, the teacher may involve the students in demonstrating that "saying 17" is a "winning theorem", or that the "Race to 20" is a "Race to 17". For doing so, he needs to interact with the students in a mathematical way, very specific to the "Race to 20" knowledge. His behavior would be "mathematically" different if the content of the lesson were a geometrical piece of knowledge.

On the other hand, we also postulate that the teacher has to use generic techniques: for example, at any time in the learning process, for any piece of knowledge, the students have to get involved in the tasks, they have to memorize the essential features of the pieces of knowledge being taught. By doing that, they have to assume the didactic contract. Some of the means that the teacher uses for that purpose are not specific at all, but pertain to the general teaching-learning process. Thus we argue that the teacher, in a constant dialectic, calls upon teaching techniques that are specific to the material being taught as well as upon generic educational techniques. In the Section 3.3 of this paper, we will show how the relations between these two types of techniques can be analyzed.

2.2.5. *Mesogenesis, topogenesis, chronogenesis*

In a broader view of the theory of didactic transposition (Chevallard, 1991, 1992; Mercier, 2002) we consider a triple dimension that describes the teacher's work, relative to starting and maintaining a didactic relationship (Sensevy et al., 2000).

Mesogenesis describes the process by which the teacher organizes a milieu, with which the students are intended to interact in order to learn. Without a specific mesogenesis action from the teacher, students play the "Race to 20" without studying any winning strategy. For example, when the teacher asks the students to write down in a table the process of their games, he brings some new constraints and resources into the learning environment, so he creates a new milieu. This is a mesogenetic action.

Topogenesis describes the process of the division of the activity between the teacher and the students, according to their potentialities. The teacher should define and occupy a position, informing students of tasks which will allow them, in turn, to occupy their positions in the didactic space. For example, in the beginning of the "Race to 20", the teacher himself may play against the students, or act as a referee, or simply observe the game. These different techniques are three different ways of dividing the didactic space in function of what each participant is supposed to know and, therefore, do. They refer to three different topogeneses. In the same vein, if a student claims that "saying 14" is a winning theorem (the student may say "it's a good number") the teacher may assume a high status position in validating this proposition (that means : "I know you are right and it is my task to tell you you are right"), or he may keep a low profile, asking the other students to react ("I know you are right, but the others have to acknowledge it, so it is not my task to validate now"). This is a topogenetic choice.

Chronogenesis describes the evolution of the knowledge proposed by the teacher and studied by the students. This progression produces, for the teacher and the students alike, a temporality that is unique to learning institutions, and that we define as the didactic time. The teacher has to monitor the knowledge process through a lesson or several lessons, in order to meet his didactical intentions. For example, in the "Race to 20", if a student claims that "saying 14" is a winning theorem, the teacher can decide not to take up this proposition, because this argument is brought too soon with respect to the "cognitive state" that the teacher infers from the other students' work. This is a chronogenetic action.

Our aim is to connect the categories we have presented in this theoretical framework, and, by doing so, to enhance their relevance in the description of the teaching-learning process. This vocabulary will be used in the analyses that follow in Section 3 of this paper. In the last part of this article, we will focus on what we consider as the background of these techniques. This means that we will try to identify, in the teachers' discourse about their teaching actions, some beliefs and values which can explain the didactic patterns they use.

2.3. *The methodology of the analysis*

In the following parts, we will describe the teacher's action using three levels of analysis:

- the interaction of mesogenesis, topogenesis and chronogenesis (3.1)
- the relationship between contract and milieu in learning games (3.2)
- the teacher's beliefs and usual ways (3.3).

In these sections, we will specify three types of monitoring the teacher can develop, differing from each other, in particular, by the scale appropriate to their description.

In the first type of monitoring (3.1), we consider that to get the students to learn, the teacher must constantly move the knowledge forward producing some didactical time (a chronogenetic constraint), then ensure a sharing of the tasks between those the teacher is responsible for and those devolved to the students (a topogenetic constraint) and manage the class's relationship with a material or cognitive milieu (a mesogenetic constraint). This triple constraint is inherent to the learning games, which the teacher must define and monitor as learning situations.

This first level of analysis is most of the time based on brief interactions, made of only a few speech-turns.

A second type of monitoring is identifiable (3.2), thanks to a medium-scaled analysis, when the teacher brings about the evolution of the learning game, as knowledge advances. By making the students connect with it, the teacher moves the didactic interaction to another goal, by another stake to the game, and thus creates another milieu and contract. In order to understand the teacher's action, we have to describe the way different learning games follow one another throughout a lesson.

This second level of analysis is grounded in interactions longer than the first one, in order to identify different goals and game structures in the learning–teaching process.

To these two types of monitoring we must add the system of beliefs among the determinants of the teacher's action. As pragmatists do, we consider beliefs as habits of action. However, if we want to identify some of the teachers' "teaching beliefs", and behaviors these could entail, we have to stop describing classroom interactions, and focus on other levels of analysis.

Thus, this third level of analysis is mainly based on interviews with the teachers interviews, and on the analyses they provide of their own actions.

3. A COMPARATIVE DESCRIPTION OF THE TWO TEACHERS' ACTIONS

For each teacher (T1 and T2) we will provide both a breakdown and a summary of the information concerning the lesson they delivered in their first classes. Then, we will focus on an episode that came from these breakdowns, and we will show examples of mesogenetic, chronogenetic and topogenetic techniques. The comparison of two "modes" of action will show how the model of the interaction between the teacher and the students is used.

3.1. *Analysis level 1: The interaction of mesogenesis, chronogenesis and topogenesis*

3.1.1. *Lesson 1 of T1*

In Table I, the bordered sections are those that will be studied further. The use of italics indicates what the teacher wrote on the board.

This lesson lasts 60 minutes, out of which 32 consist of trio or team work. First of all, the teacher asks the students to imagine what kind of game "The Race to 20" could be. Then he introduces the rules of the game (minute 6). When the list of the winning numbers comes up on the board, the teacher maintains a doubt. Michaël's statement and demonstration of the technique are given neither attention nor comments: Does his proposal come too soon in T1's lesson plan ? It is worth noticing that the teacher does not take part in the game with his students, but instead chooses to be a referee. Thus, he does not follow Brousseau's original scenario in its first phase: "The teacher explains the rules of the game and starts playing a round at the chalk-board against one of the children, then relinquishes her place to a second child" (Brousseau, 1997, p. 3). In a more general way, during his lessons, T1's instructional device is far from Brousseau's engineering. Indeed, he creates a referee function assigned to a student in the individual one-against-one game.

3.1.1.1. *Mesogenetic techniques.* From a mesogenetic point of view, the period of the lesson highlighted by the border in Table I (minutes 22–28, lines 165–192) represents a major change in the milieu. Indeed, the students were first confronted with the title "The Race to 20" and asked to imagine what it could mean. Then, they were introduced to the rules of the game and played it in groups of three, one student against another under the supervision of a third student acting as a referee. Around line 165, however, T1 moves into another stage of activity, consisting in eliciting "comments" that could be formulated in terms of mathematical features related to the

TABLE I
Breakdown of lesson 1 of teacher T1

Time in minutes	Speech turns (SpT)	Didactic episodes	Work modes
0–6	1–46	Lesson starts. Class discusses possible meanings of the title "The Race to 20". T1 explains rules of the game.	Entire class
6–11	46–113	Trial game begins, involving various students. (T1 doesn't play.)	
11–15	113–123	T1 instructs students to play in groups of 3 (1 against 1 + 1 referee). T1 explains referee's role: "Ensure that the rules are followed and keep notes of the results, possibly adding comments. Then take a turn as a player."	
15–22	124–164	Game starts in groups of three. T1 walks up and down the aisles.	Students in groups of 3
22–28	165–192	After playing, the students remark "The one who says 17 wins." T1 writes on the board: When you get to 14 you're sure to win. S1: "11." S2: "Even at 8 and also at 5 and at 2." S3: "No, he said 2 and he lost." Michaël: "That's because he didn't use the right technique. I say 2, Cédric has to say 3 or 4. Me, 5. He has to say 6 or 7 and then I'll say 8. He will have to say 9 or 10, I will say 11. He will have to say 13 or 12, I will say 14. He will have to say 16 or 15, I will say 17 and then it's over: He'll have to say 19 or 18." T1 reinforces the idea that some students who said 2 lost the game "anyway" and concludes "It would seem that some numbers are more important than others." T1, addressing Michaël, says "That was pretty clear; you got there pretty quickly."	Entire class
28–40	193–209	T1 asks to play again, being himself the referee. Following a comment from a student, T1 writes on the board: *The one who starts, wins?*	Students in groups of 3
40–48	210–327	A group plays in front of the rest of the class. Students remark: S4: "It's cheating if the same person always starts the game."	Entire class

(*Continued on next page*)

TABLE I
(*Continued*)

Time in minutes	Speech turns (SpT)	Didactic episodes	Work modes
		S5: "With the right technique, the one who starts always wins." S6: "I started but my opponent won." T1 writes on the board: *17, 14, 11, 8, 5, 2* T1 asks: "Are these numbers important? Can we say that whoever starts wins?" Students: "No." T1: "Not all of us seem to agree."	
48–60	327–357	T1 asks to play 2 against 2 + 1 referee. T1 joins a group as referee. T1 asks: "They won: was it just luck?" T1 collects what the students have written.	In groups of 2 + 2 + referee

"Race to 20." Here, the teacher takes on the work of formulation. What is at stake is no longer the game itself but the way it is played. Nonetheless, T1 remains open to the various comments the students make.

T1, minutes 22–28

165.	T1	"Did anybody write down any comments?"
166.	Student	"We had to restart the second round because the referee whispered something."
167.	T1	"So that's rather surprising, because the referee has to make sure that the rules of the game are respected, and yet it's the referee who's whispering. So it was necessary to have a referee, but there, the referee plays an unusual role, huh?"
168.	Student	"We noticed that if someone says 17, the other player can't win. The one who says 17 wins."
169.	T1	"The one who says 17 wins. Did anyone else notice anything like that?"
170.	Student	"We noticed the same thing. At one point, Arnaud didn't win because we had the same technique and we used it in each turn. When I played with Benjamin, it was sort of random, so we thought about each number, but since Arnaud didn't have the technique. . . "
171.	T1	"Is it a technique?"

Since the conjecture "17 wins" emerged rapidly, T1 decided not to evaluate it but to focus on empirical observations made by other students (speech turn 169). Some students indeed noticed "17 wins". Others put forward remarks on broken rules (e.g. adding 3 instead of 2 or 1) or on the fact that the games were very short. When T1 wrote "17" circled on the board, thus recalling the milieu represented by the object "17 wins", the students pointed out in only three speech turns that other numeric values such as 11, 14, 8, 5, and 2, also win. That "2 wins" is questioned by Quentin, who produces a practical argument.

180.	T1	"He added 3 also. Well. The rules, we'll continue to have a referee, because while playing sometimes we forget what's going on. You made a remark about the game itself,. . . that apparently 17, ... P. writes a circled 17 on the right-hand side of board. Yes, Perrine? You wanted to add. . . ?"
181.	Student	"There is also 11 and 14. When you get to 11, not when you get to 14, you're sure to win."
182.	T1	"When you get to 14, you're sure to win."
183.	Student	"Yes, but that's it. Even at 8."
184.	T1	"8 also."
185.	Student	"And also 5. Also 5 and 2."
186.	T1	"Quentin?"
187.	Quentin	"No, because I played against Hugo. He said 2 first and he lost."
188.	T1	"Ah"
189.	Michaël	"That's because he didn't use the right technique. I say 2, Cédric has to say 3 or 4. Me, 5. He has to say 6 or 7 and then I'll say 8. He will have to say 9 or 10, I will say 11. He will have to say 13 or 12, I will say 14. He will have to say 16 or 15, I will say 17 and then it's over: He'll have to say 19 or 18."
190.	T1	"Apparently, he played a round, he said 2, and he lost anyway. And if three of you speak at the same time, we'll have trouble hearing you. Right?"
191.	Student	"That's also why, there are lots of numbers that allow you to win. Not right away."
192.	T1	"Well, it seems there are some numbers that are a little more important than others, let's say. It's a difficult idea to express. That was pretty clear; you got there pretty quickly. Let's play again. But this time, the referee will be more of a secretary, taking notes on the games. The secretary will write down the numbers played. Since you've noticed some things, we can discuss them afterwards. There is the "Comments" section of your papers that you can use. Look at the paper together before starting. Keep it for now; you might need it. . . ." The students start a new game-playing phase.

T1's "Ah!" (188) could be the sign of the beginning of a debate on this question. But Michaël then proceeds (189) to outline the steps to take and explain why they're necessary. This object is apparently perceived by T1 to be "too important" to be the first formulation (see the chronogenetic dimensions, discussed next). So T1 reintroduces the debate started by Quentin and suggests to go back to the empirical milieu (190). He thus challenges Michaël's proposal. However, he asks for a referee to note down

TABLE II
Breakdown of lesson 1, teacher T2

Time in minutes	Speech turns	Didactic episodes	Work modes
0–4	1–23	T2 organizes the lesson. Lesson starts. T2 explains rules of the game and asks students to repeat the instructions.	Entire class
4–6	24–56	T2 plays a trial game with a student. T2 starts and says "1"; another student is called to take T2's place. T2: "You get the idea?"	
6–8	56–60	T2 organizes and instructs students to play in pairs, 1 against 1. T2 hands out the worksheets: T2: "You will note down who started. . . what numbers were played at each turn." T2 writes on the board: *How can I play well?* *What do I have to do to win?* T2: "You have seven minutes."	
8–17.30	60–62	game in pairs; T2 goes up and down the rows. T2: "Play as many games as possible."	In pairs, 1 against 1
17.30–19	62–82	Students remark: S1: "You have to get to 17 because then you're sure to win." S2: "You have to be careful." S3: "I do plus 2 plus 1 plus 2 plus 1." T2: "Put your worksheets aside for the moment."	Entire class
19–24	82–99	T2 organizes a game by creating a purple team and an orange team. T2 reminds students that team members need to advise their players.	
24–49	99–305	Members of the two teams play 12 games, and everyone has a turn. T2 reminds students that for each round the teams need to work together and advise their players.	
49–1h–10	305–448	T2: "We're going to think about the discoveries you've made." Students remark: S4: "One even number one odd number." T2: "How should I write that?" S5: "You have to try to get to 14." T2: "The one who says 17 is sure to win?" T2 writes what he hears on the board.	

(*Continued on next page*)

TABLE II
(*Contined*)

Time in minutes	Speech turns	Didactic episodes	Work modes
		T2: "The team that puts forward a decision everyone agrees on wins a point . . . but you have to prove it." S6: "You have to get 14." S7: "No, 15." T2: "Do we need to play a round for proof?" Round between 2 students: S8: "At 15 you're sure to lose" S9: "If you want to say 11 you have to say 8." S10: "If you want to say 8 you have to say 5." S11: "And if you want to say 5 you have to say 2." T2: "Let's verify this; who wants to play?" S12: "If you know the techniques and you start with 2 you've won." T2: "We'll write that in our notebooks." On the board: *17, 14, 11, 8, 5, 2*	

the rounds and he draws the students' attention to the "Comments" section of their papers (192): The game-playing action is thus coupled with formulation within the groups. This seems a relevant example of how mesogenetics techniques (e.g. introducing a new writing rule in the students' activity) allow the teacher to make positive "integrating choices" and negative "ignoring choices" among the students' proposals, with regard to the particular set of material and symbolic objects (the "milieu") he wants to set up.

3.1.1.2. *Chronogenetic techniques.* The same episode enables us to describe the teacher's way of managing the didactic time during such lessons. Between speech turns 182 and 186, time could suddenly have accelerated. If the teacher had instituted the proposals of the students who produced the "winning series" straight away, it would have been detrimental to the class's "cognitive state", because the key to the series is the theorem "17 wins", which had not yet been demonstrated. Introducing Quentin's contestation into the discussion enables the teacher to slow down the didactic time. T1 will ignore Michaël's contribution (192), cast doubt on it and maintain a high level of uncertainty in the class. However, the idea will be reused when "Michaël's technique" will provide the teacher with a relevant argument, at the right time, i.e. when the class is mature enough to take it. This is a good example of how a chronogenetic technique rests on the teacher, both with regard to the knowledge (he decides which proposals are interesting) and to the students (he decides whom to let speak according to the proposals he wishes to emphasize). The teacher's position is high enough to ensure he maintains control of the pace of the lesson.

3.1.1.3. *Topogenetic techniques.* This excerpt highlights the fact that T1 expresses his opinion rather indirectly: "It seems there are some numbers that are a little more important than others. . . ." The teacher does not want to emphasize the "good answers" when they are coming too soon. We are faced with a classical problem that is at the heart of a large number of research situations. The teacher wants to make the students aware of a type of knowledge (here the dialectic between mathematical necessity and the effective steps to meet this necessity), but he wants to do so without "giving the answer straight away". He therefore pretends ignorance, which can be perceived by the students as the opportunity to explore the cognitive environment (Greeno, 1991, 1994), without restricting themselves to a particular direction too quickly. This attitude will remain constant throughout the two lessons and may correspond to a didactic style that the students are used to.

In the students' and the teacher's comments we see Wittgenstein's distinction between a constraining process (terms relating, according to him, to physical determinisms) and a directing process characterizing, in particular, mathematics, in which one can always do something other than what is prescribed by rule. We can also note that when T1 speaks to Michaël to gently rebuff his argument, evoking time ("you got there rather quickly"), he undoubtedly means "too quickly for the rest of the class and therefore for the good development of the activity."

3.1.2. *Lesson 1 of T2*

During the 1 hour and 10 minutes first lesson T2 devotes 9.5 minutes to one-on-one work and the rest to group work. The stakes of the one-against-one game are announced and written on the board: From the start the students know that they have not only to win but also to explain how they did it. Very little time is devoted to observations about this one-against-one phase of the game, and, during the group game between the orange and purple teams, the team members are told they are expected to advise their players properly. This phase of the game, in teams, is the longest (25 minutes) and provides an opportunity for all the students to participate (in 12 rounds). The concluding phase (21 minutes) is also rather long: it is a time during which T2 emphasizes the students' "discoveries" and suggests them to "think about them." The conclusion is clearly to be instituted since T2 proposes that students write the final observation in their notebooks. Contrary to T1, T2' choices are very close to Brousseau's engineering (introduction, game with two groups, game of discovery).

T2, minutes 49–1 hour 10 (bordered part of Table II)

308.	T2	"(...) What have you discovered? What enabled you to win? So, we're going to write down your suggestions here. So you'll give me your proposals–proposal of discoveries. Here we'll check that? it's true–thus, a discovery accepted and verified. So does the purple team have anything to say? Who wants to start? It doesn't matter; so, Joris."
309.	Joris	"You can give one even number and one odd number, so for example, 2, the other says another number, so even, odd, even, odd."
310.	T2	"How should I write that? You noticed it, so are you sure for now? What do you have to do to win? What you've discovered, you've discovered something specific, a long while ago, Maxime?"
311.	Maxime	"Try to have certain numbers."
312.	T2	"So what did you discover?"
313.	Maxime	"You have to try to get to 14."
314.	T2	"The number 14, you say, so what about the number 14?"
315.	Maxime	"With 14 or 17 they don't have the option of saying 2 or 1, the opponent is bound to win."
316.	T2	"So you win if, how am I going to write this?"
317.	Maxime	"If you get to a certain number."
318.	T2	"But which?"
319.	Maxime	"14."
320.	T2	"If you, if you say what? 17, what happens, you're sure to win? OK? If you say 17 you're sure to win. That's one proposal. Is it accepted by the orange team?"
321.	Orange team	"Yes."
322.	T2	"When a decision is declared to be accepted, you win a point, that's how we'll do it."
323.	Student	"Who won a point there?"
324.	T2	"The purple team."
325.	Students	"Oh no!!!"
326.	T2	"But then it'll be your turn, you may have discovered something else; if someone gives a false response and you show that it's wrong you get 3 points."
327.	Student	"3, oh yeah, that's OK!"
328.	T2	"You have to prove it, of course, you have to prove it! It's the orange team's turn now."
329.	Laura	"OK, when you get to 17 you win, but to get to 17 there has to be certain numbers beforehand."
330.	T2	"Does the orange team have any ideas about that; I'll only write it down when you agree. Go ahead, Laura."
331-332		inaudible
333	Laura	"To get to 17 you have to have 14."

334	T2	"If. . . ."
335	Laura	"If you want to get to 17."
336	T2	"If you want to say 17, is that it? 17"
337	Laura	"You first have to have 14."
338	Fanny	"No, it could be 15 !!!"
339	T2	"You have to say, Laura?"
340	Laura	"14."
341	T2	"You have to say 14. OK, we'll try to check that."
342	Orange team	"Or 15."
343	T2	"Make up your mind!"
344	Orange team	"No, we say 14."
345	T2	"Do you agree or do we have to play a round to prove it?"
346	Purple team	"It's OK!"
347	T2	"Who has another opinion? (. . .) If you want to say 17 you have to say 14, do you agree?"
348.	Student	"Yes, but not too much, because you can say 15."
349.	T2	"You can say 15, so your counterproposal is that you can say 15."
350.	Student	"But you can also say 14."
351.	Student	"You can say 14 or 15!!"
352.	T2	"OK, we're going to play a round, let's start. Let's have Laura and Fanny, and we'll look for the answer together."
. . . /. . .		
362.	T2	"You said you could say 15, can you say 15 here? You said earlier that you could say 15, you're the one who said it, huh! So are you keeping 14 or will you say 15?"
363-367.	Students	(the students say in turn) "16-17-18-20"
. . . /. . .		
368.	T2	"You said you could put 15 there. Is that proposal validated here?"
369.	Student	"Yes. . . "
. . . /. . .		
441.	T2	"(. . .) so who wants to tell me what you've just learned here? What do you need to do to win the "Race to 20"? Hurry up. Amélie, you want to tell us what you've just discovered?
442.	Amélie	"The first condition is to say 2.
443.	T2	"Is it enough to say 2? What's the technique? Go ahead, Amélie. Who wants to help her? Choose someone to help you.
444.	Amélie	"Jérôme."
445.	Jérôme	"If you say 2 and you know the techniques, when the opponent answers you can say 5, after you can say 8, you can say 11, you can say 14, you can say 17 and you can say 20."
446.	T2	"And you will have?"
447.	Student	"Won!!"
448.	T2	"We'll write that down in our notebooks, I think it's right, thanks. Class is over for today."

At the 49th minute, the students were confronted with a new milieu that they had been warned of, on the basis of comments they made while playing the game.

Indeed (see Table II), the students were first given the rules of the game, then they played a trial game, and they ended up playing in pairs (1 on 1). By doing so, they encountered a milieu dedicated to action (according to Brousseau's theory), while knowing that the goal was to formulate conjectures on "how to win". The debate on discoveries was at first very brief (speech turns 62–82) and then led to a game in two teams. At this point, the first milieu dedicated to action turned to a milieu dedicated to formulation. Indeed, within the team, a strategy has to be formulated and communicated to the player representing each team.

3.1.2.1. *Mesogenetic techniques.* In the phase devoted to the debate on conjectures (game of discovery, in Brousseau's engineering), T2 dismisses the proposal concerning even and odd numbers.[5] But the decision in favour of "14 wins" or "15 wins" is not obvious because the reasoning was not applied to the case of 17, which led T2 to reintroduce the idea (320).[6]

Then, nonmathematical features, which nonetheless constitute the groundings of the didactic situation, appear in the milieu. The team that makes a proposal accepted by the other team wins one point (322) while a team succeeding in proving that the other team's proposal is false, wins 3 points (326). But it did not work. T2 (352) goes back to the milieu dedicated to action (the one-against-one game), as T1 did. T2 did not accept Laura's statement, he called it an opinion (347); the same goes for Maxime (316). Eventually, the class seems to agree that the winning discoveries in the last game have to be the winning strategies of the first game (the one-against-one "Race to 20" game corresponding to the milieu dedicated to action). The "validated" milieu thus includes the entire numerical series as well as the principle of "who starts" in conjunction with the possibility of saying 2 and necessarily winning. To end up with the lesson, T2 "sums up" the milieu, to some extent, by asking students to reiterate "what they have to do in order to win the "Race to 20": Once the discovery has been written on the board, it can be put in the students' notebooks (448) and, thus, instituted. During this phase, the teacher and the students co-elaborate new symbolic objects (discovery, validation. . . , most of the time without naming these objects) and new material objects (e.g. the writings on the board). The mesogenetic techniques enable the teacher to introduce such objects.

3.1.2.2. *Chronogenetic techniques.* The milieu described earlier is built up through interactions between T2 and the students, but the organization of the objets emerging successively, relies mainly upon the teacher. T2 does not impose an unflagging pace. It is only at the end, during the final summary of discoveries, that T2 expresses a wish that the students state them "quickly", as a final verification and as a way to confirm what they have learned. We also notice T2's wish to end this session with this shared declaration attesting the achievement of a common knowledge during the session (437–448).

442.	Amélie	"The first condition is to say 2.
443.	T2	"Is it enough to say 2? What's the technique? Go ahead, Amélie. Who wants to help her? Choose someone to help you.

We can note that these T2's speech turns may allow for a chronogenetic description, because the teacher, in asking Amélie to go ahead, prepares the institutionalization, and speeds up the didactic time. But this action may also be characterized under a topogenetic description: this time regulation is possible only because the teacher assumes a high profile in the didactic relation.

3.1.2.3. *Topogenetic techniques.* In a similar way, T2 takes positions, makes choices, and does not hesitate to press students in order to move the situation forward. The chronogenetic techniques often need the teacher to be in high topogenetic positions, and reciprocally, the topogenetic techniques are linked to the pacing of the didactic time.

When Fanny questions the conjecture "If you want to play 17 you have to play 14" (338), T2 tries to press Laura for the "right" conjecture, but the members of the orange team do not agree at all. The choice of 14 or 15 as winning numbers continues to be debated (342 and 344). After a while, because the proposal is still being questioned, T2 finally decides to have two students to play another round, including Fanny. Playing first, Fanny finds herself in the position of choosing between saying 14 or 15. She says 14, a choice that T2 points out to her (362), reminding her of her previous argument and asking her to confirm her choice ("So, are you keeping 14 or will you say 15?"). After reminding Fanny's previous position for the last time, T2 validates the proposal that "14 wins."

This example shows a technique that seems to us to be central to the teacher's work: Teachers and students are engaged, in the beginning of the episode, in a learning "game" (as a situation on which the teacher plays) that supposes an evoked relation to the milieu. The situation consists in debating proposals, either in a logical manner or by referring to a certain part of the game that can serve as a milieu for argumentation. In such a situation, where the theorem cannot be stated and proved, in order to make progress, the teacher must return to the initial situation, asking the students to replay the game, so that they and the class as a whole, can be faced again with the practical difficulty. They play, but with the precise goal of proving a conjecture through the feedback provided by an "actual" relationship to the game. We can note, here, that this mesogenetic technique (generating a milieu), consists, in fact, in actually changing the type of playing: from then on, it becomes a learning situation.

3.2. *Analysis level 2: The relationship between contract and milieu*

The chronogenetic or topogenetic techniques that we have identified, even if they can be described on relatively large scales, were shown essentially in the midst of short interactions, and they only slightly modify the general framework of the didactic relationship. Mesogenetic techniques can also often be described in the context of a brief interaction, yet a modification in the milieu can sometimes change the nature of the interaction. This means that, in the context of an apparently stable situation with a single object, the teacher changes the stakes of the situation by proposing new forms of activity. For instance, in Brousseau's engineering, beyond the first phase (called the "dialectic of action") there comes a "dialectic of formulation" when the stake is the production and diffusion of winning strategies. Then, a "dialectic of validation" appears, when the proof of the efficiency of these strategies and the study of their consistency with the piece of knowledge already acquired and instituted are proposed to be at stake.

In the two lessons we studied, we can find such changes, in which the teacher attempts to establish a new learning game in order to move the didactic time forward.

T1, during the second lesson, for example, has the students replay rounds that they kept notes on from their first lesson, in front of the whole class, in order to study them publicly.

In this case the new learning game has a specific goal. The students have to evaluate the former rounds. By taking into account the acquired knowledge, they have to determine, for example, the error, which is supposed to lead to proving or disproving the conjectures.

This new learning game requires a particular milieu. The didactic setting is constituted by rounds replayed with new knowledge.

In addition, a didactic contract is linked to this goal and this milieu. For example, the students make an attempt at providing a critical evaluation of the moves from the previous rounds.

For the teacher, these are new ventures. It is no longer a question of acting on the local activity of the students but of modifying, in the didactic plan, the very form of their interactions with the game.[7]

With this example we can identify the main features of the link between contract and milieu. The milieu can be considered as a set of objects. The students' activity is focused on these particular objects that are the "old rounds" of which the students have kept a written record. These objects are conceptual (some "theorems") as well as material (the sheets of paper used to keep note of the rounds). But we can consider these objects in another way, if we focus on the rules of action one needs to follow in order to act in the classroom. For example, a round noted on the paper can be used as an object of reminding the students of a strategy. The same round can be used as an object of evaluation, thanks to which it is possible to identify "good" and "bad" moves. So, a certain set of objects can be viewed under two different descriptions. Under the "contract" description, one is focused on the rules of actions that objects elicit and the expectations that arise from these objects in a given situation. Under the "milieu" description, one is focused on the very objects which pertain to a given situation. But the two descriptions are completing each other.

To bring the contract-milieu relationship into focus, we can observe how the expectations of the teacher T2 in the round played by Laura and Fanny (330–349 in the excerpt quoted) are not the same as the ones he had had earlier in the lesson. This example shows a technique that seems to us to be crucial to the teacher's work. Teachers and students are engaged, at the beginning of the episode, in a learning game that consists in debating proposals, either in a logical way or by using examples. The previous rounds of the game can serve as a milieu for argumentation. But this choice has its limits, which this specifc episode enables us to notice. The teacher cannot stop using examples and counterexamples. He does not succeed in bringing out and devolving a dialectic of theoretical validation.[8] So he has to go back to the starting point, having students play a round in order for them, and the class as a whole, to be faced again with the practical difficulty. He is trying to produce the demonstration of a conjecture, but with the feedback of an "actual" relationship to the milieu, in an action situation centered on *playing the game*. This is what Brousseau calls "pragmatic proof". By changing the milieu (an actual relationship rather that an evoked relationship), the teacher changes the rules of action too. He wants the students to test the pragmatic consequences of their claims (for example, when Fanny says that she can win by playing 15). The expectations embedded in the "actual" milieu generate a new contract. The students know that when playing the game, the teacher expects them to give the proof of what they have declared.

3.3. *Analysis level 3: Beliefs and usual ways of the teachers*

The third level of our analysis is devoted to studying the teachers' beliefs. Here we will sketch out some of the characteristics of the universe of beliefs evoked by T1 and T2 during their self-analysis interviews. We will emphasize aspects that prove to be directly linked to certain actions that we have identified in the two preceding levels of analysis, to show how the didactic techniques are produced on a background of beliefs whose description belongs to anthropological studies. Thus, for each teacher, some beliefs directly organize the teaching process. This points out to the need in didactic analyses to demonstrate the permeability between content specific objects and more generic educational objects across the various levels in analyzing the didactic contract.

In his management of the "Race to 20", only T1 to used the technique which consisted in having the students imagine the goal and possible rules of a game called "Race to 20." This teacher is also set apart by having placed himself in the "lowest" topogenetic position possible and by his quasisystematic failure to institute any piece of knowledge. During the self-analysis, here is what the teacher said about his techniques. The significance he attributes to these techniques led us to believe they were a deliberate choice on his part:

> (. . .) So from a word or an expression or a sentence, the setup, in fact, there, it was to try to find out how it could, what ideas that could give us about the game (. . .) I'm used to doing it that way, trying to establish a link between the greatest number of activities that apparently don't have anything to do with each other, so as to try to find out a coherence in particular areas, which is also at the origin of the idea of working from plans. It's true that in maths it's a little more difficult (. . .) The difficulty with regard to that, that's something that I frequently try to do because in my own experience, I, myself, didn't experience that coherence until very late in my studies and we worked in a 'compartmentalized' way, so I undoubtedly discovered a lot thanks to my profession. But the difficulty, in fact, we see that there are many students who aren't really in the game (. . .), who aren't interested in the questions. So. . . you can really feel there, that there's a group (. . .) indeed, there's a certain number of students who, for the moment, don't seem concerned, don't seem motivated. And the idea of questioning, it's also to motivate them a bit (. . .)

In this "explanation", we find the effect of an effort to convey coherence, rooted in the personal educational history of T1, who wishes to involve the students who are slow to get interested in the work. What this teacher says is a good demonstration of how difficult it is to associate general motivational processes with topogenetic techniques of devolution. It is likely that T1's choice of having a student act as a referee for each round of the game also comes from his wish to involve the students:

> (. . .)It was. . . so there was observation, in fact, by the referee, especially with regard to the rules of the game. So, indeed, I had to check that, yes, they really added 1 or 2 because that (. . .) so we had to understand each other, so I did not want the situation to get out of hand, we had to avoid misunderstandings because it went too fast, for example. (. . .) We do that often. For assessments, for

example, they're often associated with... either when we're doing some research on something... (*gives an example in Physical Education*) (...)

This type of thinking undoubtedly played a major role in the use of the arbitration device, since it is a frequently used organizing principle to which one can certainly attribute, beyond its particularities, a function of coherence in the varied learning of elementary education. Arbitration thus serves a double function of linking present activity to the past activities and of distancing the students from their activity through taking on "different roles." One can say that T1 seeks to involve the students in the management of the educational relationship by attributing to them circumstantial roles that allow him to balance the interaction between people: "(...) I'm certainly thinking about the taking of power, let's say of the teacher, I really want to avoid that, undoubtedly because of a past experience." We note that this seeking of symmetry is thought to be, fundamentally, a distinctive feature of all relationships. It is not due to an interaction between agents, named by an institution, which interaction would be mediated by the sharing of knowledge that one (the agent in the position of teacher) has before the other (the students in the position of those taught). This view is consistent with the low profile systematically kept by T1 throughout the lesson:

(...) It's something that I do, to play the innocent or I pretend I do not understand (...) Maybe I tend to do this too much and too often, I don't know, I like doing it (...) It sometimes causes trouble... I have students who wait for me outside the classroom (...) who don't understand the game, who are going to say to themselves, 'The teacher messed it up again'. (...) It's a big risk I take, to play with them and say stupid things and pretend not to understand or to make a student say something that seems obvious (...)

The coherence in the system of beliefs is clearly stated, including its "risky" effects. Familiar with this type of thinking, the reader will not be surprised to know that T1 "[doesn't] much like the competition aspect" and that from there comes the fact that consequently he "didn't focus much on who won." We feel that the teacher's explanations can be interpreted as follows. One of T1's fundamental beliefs lies in the need to "try to establish a link between a maximum of activities that don't, on the face of it, have anything to do with each other, so as to try and find a coherence between particular areas." From this perspective, we can interpret a large number of this teacher's techniques: We see that the technique of drawing out questions from the students at the beginning of the lesson is what we can call a devolution premise that goes beyond "the bare minimum" of all didactic interaction, in an attempt to attain larger and more ambitious objectives for the rational appropriation of knowledge. Undoubtedly this focus has biographical origins that we would need to look into more deeply. Here, the teacher exposes a general view of the use of a topogenetic technique ("playing the innocent") about which he points out its communicational complexity. We see how this technique can create didactic necessities, in which space must be given to the students in order for the didactic time to move forward as well as to the cognitive values of the teacher, for whom affecting ignorance constitutes a way of separating knowledge from the effects of imposition. Some didactic techniques (that we note in Sections 3.1 and 3.2) are truly rooted in educational beliefs.

The words of T2 are less characterized by traces of his "professional philosophy" to some extend, but his comments—made while watching the videorecording of the lesson—on certain topogenetic or chronogenetic actions revealed a lot about his didactic and educational beliefs. In relation to the beginning of the activity, T2 says, significantly, "(...) There I am, waiting for everyone's attention, and that's an instituted code. After a while everyone knows that you have to, you have to prepare yourself (...) It's something I should probably do instinctively but it is quite frequent (...)." With regard to the organization of the class into working groups, the teacher specifies "(...)I make it so that in each group there is a dynamic element, a leader who will carry the group". The teacher thus secures a possible didactic tool for moving forward the lesson by differentiating the students' positions. In particular, with regard to letting the class speak up, T2 explains how the spokespeople are selected:

> (...)I must have intended to choose a certain student so I think I let the others raise their hands and have the intention of speaking (...) I try to make them aware that when you raise your hand you've formulated your words in your head and you intend to communicate them (...) You see one hand, two, three, and little by little you do see that the students need some time (...)

Among the practices declared as "frequent" during this lesson, there is also the return to the instructions written on the board "so that those who don't remember after a while can refer to it if they're really autonomous and are used to doing so (...) I prefer to ensure that it's very visible and that it's a point of reference for the kids (...)." In the same monitoring spirit, in devolution of organizational habits, T2 says "(...) I also always give them a specific time during which they can try to achieve the objective so that they learn to manage their own time," and he adds "In groups, in our ritual, there is always someone who watches the time." We see that the words of T2, as opposed to those of T1, remain directly linked to the "Race to 20", while simultaneously revealing methods, and thus traces of a teaching style, inherent to the didactic contract established in the class.

> (...) something I do a lot; that is to say, I let them answer, I write everything on the board, and then... So, everyone has a turn... There's no direct validation,

you see. It's afterward that we take a second look at things and give counterexamples, and all the students get a chance to talk. . . , the kids who ask questions and then, in general, it's always the same ones who ask the really interesting questions (. . .)

The implication of all students in the lesson also emerges in the organization of enough games at the board to enable every student to represent his or her team ("My concern was that each student should come and represent the team).

4. Conclusion

This paper is an attempt to take into account the complexity of the teacher's action. In order to do that, we have tried to characterize this action under three different descriptions.

The first and second type of descriptions mainly allow us to determine the effects of the didactic constraints on the teacher's behavior in the classroom. These two types of descriptions are only differentiated by the scale of the analysis. The third type of description consists in bringing the educational background of these techniques into focus.

It is crucial to point out that these three levels are interconnected. We think that the quality of description of the teacher's action depends on how we manage to show the three levels interweaving.

Let us try to manage such a threefold description on an example.

As we saw above, T1, during his first lesson, has to deal with an early declaration of a student (Michaël, in 189).

182.	T1	"When you get to 14, you're sure to win."
183.	Student	"Yes, but that's it. Even at 8."
184.	T1	"8 also."
185.	Student	"And also 5. Also 5 and 2."
186.	T1	"Quentin?"
187.	Quentin	"No, because I played against Hugo. He said 2 first and he lost."
188.	T1	"Ah"
189.	Michaël	"That's because he didn't use the right technique. If I say 2, Cédric has to say 3 or 4. Me, 5. He has to say 6 or 7 and then I'll say 8. He will have to say 9 or 10, I will say 11. He will have to say 13 or 12, I will say 14. He will have to say 16 or 15, I will say 17 and then it's over: He'll have to say 19 or 18."
190.	T1	"Apparently, he played a game, he said 2, and he lost anyway. And if the three of you speak at the same time, we'll have trouble hearing you. Right?"

At the first level, the sharpest, we can point out how the three meso, chrono-, topo- genesis techniques work together to sow a doubt in the class.

When T1 interacts through speech turn 190, there are already many objets brought into the milieu by the students. Then, the mesogenetic technique consists in selecting and indicating one object (put forward by Quentin) in order to enlarge the milieu, dangerously shrunk by Michaël's proposal. By choosing Quentin's objection, the teacher emphasizes a contradictory proposal and behaves as if he didn't know more than the students.

Therefore, at this moment, he positions himself within the same didactical space as the students. This is a topogenetic technique, that enables doubt to be sustained.

Furthermore, the teacher uses a chronogenetic technique, intended to slow down the didactical time by jumping backward on a previous proposal (187), prior to *Michaël's* one. Through this example, the narrow intertwining of the first level techniques is revealed.

Enlarging the description scale from minutes 15 to 28, at the second level of description, we understand that the increased uncertainty in the class leads to a modification of the learning game. After the speech turn 191, continuing the same discussion could prove counterproductive. Indeed, there would be no means to validate or invalidate the proposals. So, a new round has to be suggested in order to reduce uncertainty (in which "The secretary will write down the numbers played, since you've noticed some things, we can discuss them afterward"; see above 192). This turns out to be a new game with slightly modified rules. Therefore it can be described as a new milieu and a new contract. In this type of description, we can consider the teacher's utterance as a means to create uncertainty, and by doing so, to create the necessity to reduce this uncertainty by introducing a new learning game.

One should note that these two types of descriptions are "effect descriptions". In describing the teacher's action with a peculiar vocabulary (mesogenesis, chronogenesis, topogenesis, milieu, contract. . .) we show how the teacher's action is constrained by the knowledge. The linguistic form of these descriptions could be generally paraphrased as follows: in order to teach pieces of knowledge, the teacher has to respect didactic constraints, and make use of related techniques.

At the third level of description, the teacher's utterance appears to be based on a general belief. As we saw above, for T1, making use of the "playing the innocent" technique is in harmony with one of his educational aims. It seems to be crucial, to T1, to give some didactic space to the students to avoid effects of authority. Hence, this third type of description is no longer an "effect-description": the teacher's behavior is studied in the frame of more general habits, which are not directly produced by the didactic constraints. It does not mean that these general habits are disconnected from the "didactic" ones. One could suppose, for example, that the "playing the innocent" technique is an important one for this teacher because it has both didactic effects and educational (in a broad sense) effects. The relevance of the techniques should be evaluated under these two ranges of descriptions.

According to us, this example seems to constitute a fundamental dialectic of the teacher's work. One part of the determinants of the teacher's action is to be found in the inherent constraints of the structures of the didactic relationship; another part is identified in the beliefs developed through the contact with these constraints, beliefs converted into habits of action, into pragmatic matrices, like the "playing the innocent" technique. These two systems of determination are in constant interaction: habits of action are continuously redefined under the constraints of the didactic processes, which can themselves be displaced as the action unfolds.

The attempt to produce such threefold descriptions has some methodological consequences.

First, the researchers have to collect data both on the teaching process itself and about the teacher's beliefs. In order to do that, the self-analysis, by the teacher, of his performance is very useful.

Secondly, as we did in this research, it seems relevant to elaborate methodological devices in which both the experimental and the clinical dimensions are included. We conjecture that the "Race to 20" experimentation elicited some essential techniques that it would be more difficult to isolate in ordinary lessons. The "Race to 20" seems to be relevant in showing a range of techniques linked with the moves from one phase to another ("action" to "formulation" then to "validation") through the appropriate games. At the same time, it seems to us that the teaching conditions were ordinary enough to preserve the "ecological validity" of our findings. However, this will have to be confirmed through further work, on other mathematical situations, involving different types of knowledge.

The research described in this article is furthered in the following two directions which seem very important for us to get a deeper understanding of the dynamics of the teaching-learning process:

- Enabling us to apply the same system of categories to the description of an ever-growing number of situations. By doing so, one can hope to make useful comparisons between the teachers' techniques in almost every ordinary situation, the components of which are determined by an a priori analysis.
- Developing a teacher training unit about the teacher's action in the so-called "investigative activities" in mathematics classes. It is a question of documenting the extremely technical nature of the actions a teacher must undertake in an adidactic situation. Our aim is to create thus the

conditions to transform general teaching techniques (so often remarkably well mastered by teachers) into specific mathematics teaching techniques and, in doing so, to grant them their true scope.

Notes

1. The gender of the teacher was not a variable in our research and we will not disclose it here. We will use the generic masculine pronouns instead of compounds such as "s/he" merely for the sake of alleviating the text.
2. The player who says 17 can control the progress of the game, saying 2 if the opponent says 1 and 1 if the opponent says 2. So a turn in the game makes 3.
3. The game "Race to A" is therefore equivalent to the game "Race to R".
4. Most teachers who tried the situation never connected it with the Euclidean division during the teaching process. It requires to then play the Race to 30 by adding 1, 2 or 3 . . . etc. Even in this research, where the teachers were trained with the mathematical meaning of this situation, the Euclidean division did not emerge during the teaching process. The expected model is obtained from the perspective of equivalence of games conceived as mathematical structures, not from the perspective of finding a strategy of winning in real games of a certain type, which is most certainly the perspective of the player, and to a certain extent, of the teacher.
5. During the self-analysis interview, P2 will say about this "Yes, I eliminate that because (. . .) I think that it could have made us lose track of what we were observing".
6. In the post-lesson interview P2 clarifies, "She (a student) picks up on isolated pieces of information, without taking other information into account. . . ."
7. The effort fails, however. The teacher does not succeed in installing a new dialectic that will meet the requirements necessary to the evolution of the students' problem. At the end of the study, the students were nonetheless able to play a trial game (on the whole correctly) of the "Race to 30", with a common difference of 3.
8. The teacher has not instituted a strong demonstration of the proposal "17 wins against all defence." He could have reused this demonstration, which would have formed the basis of/for the notion of a "winning strategy."

References

Brousseau, G.: 1997, *Theory of Didactical Situations in Mathematics*, Kluwer Academic Publishers, Dordrecht.

Chevallard, Y.: 1991, *La transposition didactique*, La Pensée Sauvage, Grenoble.

Chevallard, Y.: 1992, 'Fundamental concepts in didactics: perspectives provided by an anthropological approach', in R. Douady and A. Mercier (eds.), *Research in Didactique of Mathematics*, Selected Papers, Grenoble: La Pensée Sauvage, pp. 131–168.

Greeno, J.: 1991, 'Number sense as situated knowing in a conceptual domain', *Journal for Research in Mathematics Education* 22, 170–218.

Greeno, J.: 1994, 'Some further observations of the environment/model metaphor', *Journal for Research in Mathematics Education* 25, 94–99.

Leutenegger, F.: 1999, Construction d'une clinique pour le didactique. *Une étude des phénomènes temporels de l'enseignement. Recherches en didactique des mathématiques* 20(2), 209–250.

Mercier, A.: 1988, Didactical contract: permanent clauses and local breaches, *Proceedings of ICME 6*, Budapest.

Mercier, A., Sensevy, G. and Schubauer-Leoni, M-L.: 2000, 'How social interactions within a class depend on the teacher's assessment of the various pupils' mathematical capabilities, a case study', *Zentralblatt fur Didaktik der Mathematik, International Review of Mathematics Education* 32, 126–130.

Mercier, A.: 2002, 'La transposition des objets d'enseignement et la définition de l'espace didactique, en mathématiques', Note de synthèse, *Revue Française de pédagogie* 141, 135–171.

Mercier, A., Schubauer-Leoni, M-L. and Sensevy, G. (eds.): 2002, 'Vers une didactique comparée', *Revue Française de pédagogie* 141, 5–16.

Schubauer-Leoni, M.L. and Leutenegger, F.: 2002, Expliquer et comprendre dans une approche clinique/expérimentale du didactique ordinaire. In Leutenegger, F. and Saada-Robert, M. (eds.), *Expliquer et Comprendre en Sciences de l'Education*, Paris, Bruxelles: De Boeck, pp. 227–251.

Sensevy, G., Mercier, A. and Schubauer-Leoni, M-L.: 2000, 'Vers un modèle de l'action didactique du professeur. A propos de la Course à 20', *Recherches en Didactique des Mathématiques* 20, 263–304.

Sensevy, G., Mercier, A. and Schubauer-Leoni, M-L.: 2002, 'A model for examining teachers' didactic action in mathematics, the case of the game "Race to 20"', *Proceedings of European Research in Mathematics Education II*, Vol. II, Marienbad, February 24–27, 2001, 420–433.

GÉRARD SENSEVY
CREAD Rennes 2/IUFM de Bretagne
E-mail: gerard.sensevy@bretagne.iufm.fr

MARIA-LUISA SCHUBAUER-LEONI
FPSE Genève
E-mail: schubauer.leoni@bluewin.ch

ALAIN MERCIER
INRP
E-mail: mercier@inrp.fr

FLORENCE LIGOZAT
FPSE Genève
E-mail: florence.ligozat@pse.unige.ch

GÉRARD PERROT
IUFM de Bretagne
E-mail: mogeper@free.fr